The Persistence of Volatility and Stock Market Fluctuations

By JAMES M. POTERBA AND LAWRENCE H. SUMMERS*

Explaining the dramatic intertemporal variation in real stock market prices is a fundamental problem of financial economics. Between 1960 and 1985, a relatively tranquil period by historical standards, the Dow Jones Industrial Average measured in 1985 dollars varied from 3385 (June 1966) to 909 (June 1982).[1] As the analysis of William Brainard, John Shoven, and Laurence Weiss (1980) suggests, it is difficult to account for the movements in the stock market during the last twenty years on the basis of changes in either expected cash flows or real interest rates. Robert Shiller (1981) provides significant evidence that stock market volatility in general cannot be explained by movements in the rational expectation of future dividends and interest rates.[2] A natural alternative hypothesis is that market movements reflect in substantial part changing risk premia induced by movements in stock market volatility. This explanation has been suggested by Burton Malkiel (1979) and Robert Pindyck (1984) in connection with the significant decline in real stock market values between the mid-1960's and the early 1980's. They argue that changes in risk are responsible for a significant part of the variation in share prices.

This paper evaluates the changing risk premium hypothesis and examines the influence of changing stock market volatility on the level of stock prices. We find that shocks to volatility decay rapidly and therefore can affect required returns for only short intervals. This implies that volatility shocks can have only a small impact on stock market prices, and makes us skeptical of the claim that volatility-induced fluctuations in risk premia account for much of the observed variation in these prices.

The paper is divided as follows: Section I clarifies the theoretical relationship between return volatility, required rates of return, and the level of share prices. Section II examines the time-series properties of stock market volatility using daily return data on the Standard and Poor's Composite Stock Index for the period 1928–84. The results suggest that while volatility is serially correlated, changes in current volatility have relatively small effects on volatility forecasts over even short horizons. Section III uses a different type of data, the implied volatilities in option premia, to examine the persistence of volatility shocks. The results again suggest that these shocks are only weakly serially correlated. The concluding section discusses the implications of our results for alternative explanations of recent stock market movements.

I. Volatility, Required Returns, and Stock Price Fluctuations

This section explores the relationship between changes in stock market return volatility and changes in the level of stock market prices. We assume that firms are not levered and that expected dividends grow at a constant rate. The former assumption allows us to ignore Fischer Black's (1976) important observation that the level of share prices, by

*Departments of Economics, MIT, Cambridge, MA 02139 and Harvard University, Cambridge, MA 02138, respectively. We are indebted to David Cutler, Ignacio Mas, and Carla Ponn for research assistance, and to Fischer Black, Olivier Blanchard, Zvi Griliches, Greg Mankiw, Robert Merton, Daniel Nelson, Julio Rotemberg, and two referees for helpful suggestions. Financial support was provided by the National Bureau of Economic Research and the National Science Foundation. This research is part of the NBER's research program in Financial Markets and Monetary Economics.

[1] This calculation employs monthly averages of the daily closing values of the Dow Jones Industrial Average, adjusted to December 1985 prices using the Urban Worker Consumer Price Index.

[2] Shiller's claim that stock prices are too volatile has sparked substantial controversy. Allan Kleidon (1985) and Terry Marsh and Robert Merton (1986) dispute his claim, while N. Gregory Mankiw, David Romer, and Matthew Shapiro (1985) provide supporting evidence.

affecting the degree of leverage, should have a direct impact on volatility. The latter assumption is maintained for convenience and could be relaxed easily. To parallel our empirical work, we use a discrete time formulation.

We assume that share prices satisfy the standard requirement that

$$(1) \qquad \frac{E_t(P_{t+1}) - P_t}{P_t} + \frac{D_t}{P_t} = r_t + \alpha_t,$$

where r_t denotes the risk-free real interest rate, α_t the equity risk premium, and D_t the dividend paid in period t. Equivalently, equation (1) can be written

$$(2) \qquad P_{t+1} = (1 + r_t + \alpha_t)P_t - D_t + \varepsilon_t P_t,$$

where $\varepsilon_t = (P_{t+1} - E_t[P_{t+1}])/P_t$ is a random disturbance assumed to be uncorrelated with any information available at time t. It reflects revisions in expectations about future values of D, α, and r which take place between t and $t+1$.

Equation (2) can be solved forward subject to an appropriate transversality condition to yield an expression for P_t:

$$(3) \qquad P_t = E_t\left[\sum_{j=0}^{\infty} R_{t+j} D_{t+j}\right]$$

$$\text{where } R_{t+j} = \prod_{i=0}^{j} (1 + r_{t+i} + \alpha_{t+i})^{-1}.$$

Assuming that the real risk-free rate is constant, this expression may be linearized around the mean value of the risk premium $\bar{\alpha}$ to obtain

$$(4) \qquad P_t = \sum_{j=0}^{\infty} \frac{E_t[D_{t+j}]}{(1 + r + \bar{\alpha})^{j+1}} + \sum_{j=0}^{\infty} \frac{\partial P_t}{\partial \alpha_{t+j}} \left(E_t[\alpha_{t+j}] - \bar{\alpha}\right),$$

where

$$(5) \qquad \partial P_t / \partial \alpha_{t+j} = -(1 + r + \bar{\alpha})^{-j-2} \times \sum_{k=0}^{\infty} \frac{E_t(D_{t+j+k})}{(1 + r + \bar{\alpha})^k}.$$

Equation (4) expresses current stock prices as a linear function of expected future risk premia. If expected dividends grow at a constant rate g, so that $E_t[D_{t+j}] = (1+g)^j D_t$, then (5) simplifies to

$$(6) \qquad \frac{\partial P_t}{\partial \alpha_{t+j}} = \frac{-D_t(1+g)^j}{(1 + r + \bar{\alpha})^{j+2}} \times \sum_{k=0}^{\infty} \frac{(1+g)^k}{(1 + r + \bar{\alpha})^k} = \frac{-D_t(1+g)^j}{(1 + r + \bar{\alpha})^{j+1}(r + \bar{\alpha} - g)}.$$

We now consider the link between return volatility and the equity risk premium. Robert Merton (1973, 1980) derives a linear relationship between the equity risk premium, α_t, and the variance of equity returns, σ_t^2:

$$(7) \qquad \alpha_t = \gamma \sigma_t^2.$$

The parameter γ is a function of investors' coefficients of relative risk aversion.[3] To study the effect of volatility changes on P_t, we must specify the evolution of σ_t^2. Our empirical work suggests that for the postwar period, monthly values of σ_t^2 follow an AR(1) process:

$$(8) \qquad \sigma_t^2 = \rho_0 + \rho_1 \sigma_{t-1}^2 + \mu_t.$$

It immediately follows that monthly values

[3] In a one-consumer economy, γ equals the consumer's relative risk aversion. More generally, it is a weighted average of different consumers' relative risk aversions.

of α_t also follow an AR(1) process:

$$(9) \qquad \alpha_t = \gamma\rho_0 + \rho_1\alpha_{t-1} + \eta_t,$$

where $\eta_t = \gamma\mu_t$. The mean value of α_t is therefore $\bar{\alpha} = \gamma\rho_0/(1-\rho_1)$, and $(\alpha_t - \bar{\alpha})$ evolves according to

$$(10) \qquad \alpha_t - \bar{\alpha} = \rho_1(\alpha_{t-1} - \bar{\alpha}) + \eta_t.$$

From equation (10) we know that $E_t(\alpha_{t+j} - \bar{\alpha}) = \rho_1^j(\alpha_t - \bar{\alpha})$. This may be used along with equation (6) to simplify (4):

$$(11) \quad P_t = \sum_{j=0}^{\infty} \frac{E_t(D_{t+j})}{(1+r+\bar{\alpha})^{j+1}} - \sum_{j=0}^{\infty} \frac{D_t(1+g)^j\rho_1^j(\alpha_t - \bar{\alpha})}{(1+r+\bar{\alpha})^{j+1}(r+\bar{\alpha}-g)}$$

$$= \frac{D_t}{r+\bar{\alpha}-g} - \left[\frac{1}{1+r+\bar{\alpha}-\rho_1(1+g)}\right] \times \left[\frac{D_t}{r+\bar{\alpha}-g}\right](\alpha_t - \bar{\alpha})$$

$$= \bar{P}_t + \frac{dP_t}{d\alpha_t}(\alpha_t - \bar{\alpha}).$$

The last expression shows the effect of risk premia shocks on share prices. Using the fact that $dP_t/d\sigma_t^2 = (dP_t/d\alpha_t)(d\alpha_t/d\sigma_t^2)$, we know

$$(12) \quad \frac{dP_t}{d\sigma_t^2} = \frac{-\gamma}{[1+r+\bar{\alpha}-\rho_1(1+g)]} \times [D_t/(r+\bar{\alpha}-g)].$$

Multiplying both sides of (12) by σ_t^2/P_t yields

$$(13) \quad \frac{d\log P_t}{d\log\sigma_t^2} = \frac{-\gamma\sigma_t^2\lambda_t(r+\bar{\alpha}-g)^{-1}}{[1+r+\bar{\alpha}-\rho_1(1+g)]}$$

where λ_t is the dividend yield, D_t/P_t. Since $\gamma\sigma_t^2 = \alpha_t$ and $\lambda_t = r_t + \alpha_t - g$, the derivative in (13) can be written as[4]

$$(14) \quad \frac{d\log P_t}{d\log\sigma_t^2} = \frac{-\bar{\alpha}}{[1+r+\bar{\alpha}-\rho_1(1+g)]}.$$

The absolute value of this derivative rises with ρ_1. If increases in volatility are expected to persist, they will have a greater impact on the discount factors applied to future cash flows and therefore on current share prices.

To gauge the possible influence of volatility on share prices, we evaluate (14) at some plausible parameter values. Roger Ibbotson (1984) reports that the mean annual nominal return on common stocks for the period 1948–83 was 11.6 percent, or .964 percent per month. The mean nominal return on Treasury bills was 4.6 percent per year, implying $\bar{\alpha} = .006$ per month. The average real return on Treasury bills (r) was 0.4 percent per year or 0.035 percent per month. The estimated variance of market returns, expressed at monthly rates, ranged from .00022 in 1964 to .0053 in 1974 and averaged .0017. The last statistic, in conjunction with the estimate for the mean risk premium, implies $\gamma = 3.5$.[5] The annual growth rate of nominal dividends on the Standard and Poor's Composite Index during the 1948–83 period was 5.2 percent. Combining this with our annual inflation rate of 4.2 percent yields an average real annual dividend growth rate (g) of 0.01, or 0.00087 per month.

The effect of changes in volatility on the level of share prices is very sensitive to the level of ρ_1, the serial correlation in monthly volatility. The derivative in (14) equals $-.006$ when $\rho_1 = 0$, $-.012$ when $\rho_1 = .5$, $-.108$ when $\rho_1 = .9$, and $-.387$ when $\rho_1 = .99$. Stated another way, a doubling of volatility from its average level reduces the value of the market by only 0.6 percent if $\rho_1 = 0$, by 1.2 percent if $\rho_1 = .5$, and by 38.7 percent if $\rho_1 = .99$. These calculations show that if fluctuations in volatility are to play a significant role in explaining stock market fluctua-

[4]We obtain (14) by evaluating (13) at the mean return, dividend yield, and risk premium.

[5]Merton (1980) estimates γ to be 3.2 using data for the postwar period.

TABLE 1—AUTOCORRELOGRAM OF MONTHLY VOLATILITY SERIES

Lag Length	1950–84		1930–84	
	Autocorrelation	Partial Autocorrelation	Autocorrelation	Partial Autocorrelation
1	0.571 (0.040)	0.571 (0.040)	0.730 (0.027)	0.730 (0.027)
2	0.384 (0.045)	0.086 (0.049)	0.500 (0.031)	0.042 (0.035)
3	0.277 (0.047)	0.041 (0.049)	0.459 (0.029)	0.229 (0.034)
4	0.219 (0.048)	0.042 (0.049)	0.493 (0.029)	0.169 (0.033)
5	0.224 (0.048)	0.093 (0.049)	0.454 (0.030)	0.030 (0.034)
6	0.185 (0.048)	0.003 (0.049)	0.478 (0.029)	0.215 (0.032)
7	0.148 (0.048)	0.003 (0.049)	0.493 (0.029)	0.079 (0.033)
8	0.162 (0.048)	0.069 (0.049)	0.467 (0.029)	0.059 (0.033)
9	0.169 (0.048)	0.047 (0.049)	0.412 (0.030)	−0.008 (0.033)
10	0.174 (0.048)	0.038 (0.050)	0.414 (0.030)	0.069 (0.033)
11	0.132 (0.049)	−0.027 (0.050)	0.426 (0.030)	0.044 (0.032)
12	0.107 (0.049)	−0.003 (0.050)	0.419 (0.030)	0.029 (0.033)
13	0.109 (0.049)	0.027 (0.050)	0.369 (0.031)	−0.051 (0.033)
14	0.059 (0.048)	−0.049 (0.049)	0.326 (0.032)	−0.058 (0.033)
15	0.072 (0.048)	0.029 (0.048)	0.287 (0.032)	−0.067 (0.033)
16	0.065 (0.048)	−0.005 (0.049)	0.300 (0.032)	0.029 (0.033)
17	0.067 (0.048)	0.014 (0.049)	0.317 (0.032)	0.029 (0.033)
18	0.055 (0.048)	−0.015 (0.049)	0.282 (0.033)	−0.076 (0.033)
19	0.030 (0.048)	−0.025 (0.049)	0.246 (0.033)	−0.007 (0.033)
20	0.002 (0.048)	−0.034 (0.049)	0.251 (0.033)	0.020 (0.033)
21	−0.015 (0.048)	−0.024 (0.049)	0.242 (0.033)	−0.004 (0.033)
22	0.017 (0.048)	0.049 (0.049)	0.264 (0.033)	0.110 (0.032)
23	0.019 (0.048)	−0.003 (0.049)	0.295 (0.032)	0.071 (0.033)
24	0.010 (0.048)	−0.010 (0.049)	0.269 (0.033)	−0.021 (0.033)

Note: Monthly volatility estimates are calculated as the average value of squared daily returns on the Standard and Poor's Composite Stock Index. Standard errors are shown in parentheses.

tions, then ρ_1 must be quite large. The next two sections present estimates of this parameter.

II. Estimates of Serial Correlation in Market Volatility

This section examines the persistence of volatility over the 1928–84 period as well as the shorter postwar period studied by Pindyck. We use volatility estimates computed from daily returns on the Standard and Poor's (S&P) Composite Index, which were kindly provided to us by Kenneth French, G. William Schwert, and Robert Stambaugh.[6] The variance estimator for month t, $\hat{\sigma}_t^2$, is

$$\hat{\sigma}_t^2 = \sum_{i=1}^{K_t} \tilde{s}_{t,i}^2 / k_t, \tag{15}$$

where $\tilde{s}_{t,i}$ is the market return on the ith day of month t, measured by the percentage change in the S&P Composite Index, and k_t is the number of trading days in month t.

The autocorrelogram and partial autocorrelogram for the estimated volatility series are shown in Table 1. The first panel corresponds to the post-1950 period as in Pindyck, while the second utilizes data from the full sample period. The data clearly exhibit positive serial correlation, although the persistence of volatility is relatively unimportant from an economic perspective. For the postwar sample, the monthly first-order autocorrelation coefficient is .571. The partial

[6] An earlier version of this paper used daily stock market returns for the 1968–84 period, as well as monthly returns for the 1871–1984 period, to investigate volatility persistence.

autocorrelogram shows only one statistically significant value at lags higher than one month, that at lag 5, and suggests that an AR(1) representation provides an adequate description of the data.

The volatility data for the full sample period display somewhat greater persistence. The first-order autocorrelation coefficient is 0.730, and the autocorrelogram decays much more slowly than that for the postwar period. The difference in persistence is primarily due to the presence of the Great Depression years in our 1930–84 sample. As Robert Officer (1973) observes, the volatility of market returns was much higher during the Great Depression than at any time in the last century. Moreover, since volatility was high for several consecutive months, these few observations have a significant impact on the estimated serial correlation parameters. It is far from clear that the persistence of volatility during the Great Depression provides a useful guide to the current persistence beliefs of market participants.

Our analysis of the autocorrelation properties of the level of volatility implicitly assumes that volatility is a stationary series. French, Schwert, and Stambaugh (1985), however, argue that volatility is actually nonstationary and fit ARMA models to monthly differences in volatility.[7] Following David Dickey and Wayne Fuller (1981) and Fuller (1976), we explicitly tested the hypothesis that measured volatility is a nonstationary series, that is, that there is a unit root in its autoregressive representation. For both sample periods, the null hypothesis of nonstationarity was rejected in favor of the alternative hypothesis of stationarity at very high confidence levels. For example, when we test the unit root hypothesis allowing for a relatively general AR(12) specification, the test values are -3.70 and -3.97 for the 1930–84 and 1950–84 samples, respectively, while the .01 critical values are -3.46 and -3.44. Similar results obtain for a wide range of different autoregressive lag lengths.[8]

Even if volatility is modeled as a nonstationary series, volatility shocks are not highly persistent. The results of estimating an IMA(1,3) process, as in French et al., for the full sample period are

$$\Delta\sigma_t^2 = (1 - \underset{(.038)}{.404}L - \underset{(.039)}{.327}L^2 - \underset{(.035)}{.048}L^3)\varepsilon_t.$$

Three months after a volatility shock, the level of volatility is elevated by only .221 times the initial shock. Although volatility remains at this new higher level forever, the rapid decay implied by the moving average parameters makes the net effect on share prices relatively small.

To illustrate what our results imply about the impact of volatility shocks on share prices, we used equation (4) to evaluate $d \log P_t / d \log \sigma_t^2$ for several different stochastic specifications of the volatility process. We use (*i*) an AR(1) process, which as noted above describes our data reasonably well; (*ii*) an AR(12) process designed to capture long-run persistence which might be poorly characterized by an AR(1) model; and (*iii*) the IMA(1,3) model described above. In each case we compute the impulse response functions for the level of volatility, and use (4) to calculate the change in share prices associated with each of these perturbations to the volatility path.

The resulting calculations, along with the standard errors of the estimated elasticities,[9] are shown in Table 2. The impact of a volatility shock on share prices is extremely

[7] French et al. model the square root of volatility, and in a revised version of their paper, the logarithm of volatility, as an IMA(1,3) process.

[8] Schwert (1985) suggests that Dickey-Fuller tests may yield spurious results if the time-series process is misspecified, although this problem is reduced when long autoregressive processes are considered. We therefore conducted unit root tests assuming up to AR(24) processes for volatility, and rejected the unit root hypothesis in all cases.

[9] We calculate the asymptotic variance of our elasticity estimates by computing numerical derivatives of these elasticities with respect to each of the estimated time-series parameters, stacking these derivatives in a column vector f, and then evaluating $f'\Omega f$ where Ω is the variance-covariance matrix for the parameter estimates.

TABLE 2—ESTIMATES OF ELASTICITY OF SHARE PRICES WITH RESPECT TO VOLATILITY

Estimation Period and Volatility Process	$d \log P_t / d \log \sigma_t^2$
1950–84	
AR(1)	−0.014
	(0.001)
AR(12)	−0.020
	(0.006)
IMA(1,3)	−0.175
	(0.038)
1930–84	
AR(1)	−0.022
	(0.002)
AR(12)	−0.048
	(0.010)
IMA(1,3)	−0.225
	(0.026)

Notes: Asymptotic standard errors are shown in parentheses. Each entry shows the percentage change in stock market prices from a 1 percent change in stock market volatility, calculated as described in the text.

small, regardless of the stochastic specification. For the postwar period, the AR(1) process implies an elasticity of share prices with respect to volatility of −.014, while the AR(12) specification implies −.020. Both models suggest that volatility shocks have very small effects on share prices. The moving average specification for volatility differences raises the elasticity to −.175, but even this is an economically small effect. For the full sample period, all of the estimates are somewhat larger. In the AR(1) and AR(12) cases, the elasticities are −.022 and −.048, while in the IMA(1,3) case, the elasticity rises to −.225. A 50 percent increase in market volatility, even using this extreme estimate of volatility persistence, would therefore depress share prices by only 11 percent. These results cast serious doubt on the view that changes in volatility, through their influence on investors' risk premia, have a substantial effect on stock market values.

One potential difficulty with our results is that they describe the persistence of *measured* market volatility, which is a noisy observation on true volatility in any month. It is possible, although we believe unlikely, that market participants have information which enables them to determine the true level of volatility. In this case, the measurement error in our data will bias our estimates toward understating true persistence.

It may be of some interest to analyze the implied persistence in the "true" volatility series. Assuming that the measurement error is independent and identically distributed over time with a zero mean and variance σ_ν^2, and that true volatility follows an AR(1) process, then our estimate of ρ_1 will have a probability limit of $\theta\rho_1$, where $\theta = \text{Var}(\sigma_t^2)/[\text{Var}(\sigma_t^2) + \sigma_\nu^2]$. We estimate θ, the ratio of signal variance to total variance in measured volatility, to be .816 for the postwar period and .890 for the 1930–84 period.[10] These estimates imply that the monthly first-order autocorrelation coefficient for the true volatility process is .700 for the postwar period and .820 for the full sample. Thus, even if market participants could observe true volatility, volatility shocks would have a small effect on market prices.

The general finding that emerges from this section is that stock market volatility is not highly persistent. The data provide little support for the hypothesis that changes in volatility could have an important effect on the level of stock prices. How, then, does Pindyck reach an opposite conclusion using similar data? First, he constructs and graphs a moving average of monthly volatility estimates. Because averaging induces serial correlation, these estimates exhibit substantial persistence. Second, he regresses excess stock returns on the change in his moving average volatility measure, finds a statistically significant effect, and concludes that volatility shocks played an important part in depressing the stock market.

Pindyck's regression of excess returns on changes in volatility does not demonstrate that autonomous volatility changes have had an important effect on market prices. First,

[10] We calculate the noise-to-signal ratio relying on the fact that the sampling variance of the estimated variance from a normal population is $2\sigma^4/N$, where N is the sample size and σ^4 is the square of the true variance. Using $2\sigma^4/k_t$ as an estimate of each month's measurement error variance, we compute the fraction of the total time-series variation in market volatility which should be accounted for by these errors and use this to calculate θ.

since changes in his volatility measure can be predicted, his regression implies that investors should be able to realize substantial profits by trading on past volatility information. Second, the results are contaminated by a serious simultaneity problem. A decline in the stock market should increase volatility through the leverage effect discussed by Black (1976) and Andrew Christie (1982). Finding that movements in the market are negatively correlated with the volatility measure by no means demonstrates the causal role of volatility shocks in depressing the market. Even if volatility changes had no effect on stock prices, one would expect a negative correlation between returns and volatility changes. Third, the implied relationship between volatility shocks and stock returns is weak. French et al. find that the elasticity of share prices with respect to *unanticipated* volatility shocks is at most -0.0272.[11]

III. Estimates of Serial Correlation in Volatilities Implied by Option Premia

Although the estimates presented in the previous section suggest that volatility shocks are short-lived, they might fail to reflect market participants' beliefs about volatility persistence. The previous estimates also provide little guidance concerning how far back into history investors look in judging the persistence of volatility. Since the volatility process estimated for the postwar period displays substantially less persistence than that for the full sample period, this issue is of some consequence for analyzing how volatility shocks affect share prices.

To address these problems, we analyze the persistence of changes in *ex ante* market volatilities as inferred from option premia. These data represent a market estimate of the *ex ante* volatility which theory says should affect required returns and stock prices. Unfortunately, options on stock market indices such as the Standard and Poor's 500 have been traded for too short a period to make analyzing them informative. However, the Chicago Board Options Exchange (CBOE) has computed an index of the price of a standardized stock option on every Thursday since January 8, 1976:

> ...the CBOE Call Option Index is an average of percent option premiums; for each CBOE underlying stock, a market premium is estimated for a hypothetical six-month, at the money option using the market premiums of existing option series. This estimated market premium is expressed as a percentage of the stock price. The CBOE Call Option Index for a given day is the arithmetic average of all such percent premiums on CBOE underlying stocks on that day. [CBOE, 1979, p. 1]

These data may be used to analyze the persistence of volatility expectations.

The CBOE Index does not correspond to the option premium of any traded security. It is a measure of the option premium on the "representative share" for which options are traded on the CBOE. The implied volatility should therefore be substantially higher than the volatility of the market, since the market is a weighted average of many imperfectly correlated shares. While our estimates of the implied volatility on a representative share are not directly comparable to the volatilities estimated in the last section, their serial correlation properties should be similar.[12] This is supported by our finding that movements in the implied volatilities cohere reasonably well with those of *ex post* volatilities estimated from daily returns on the S&P 500.

To estimate the volatility of the "representative stock" implied by the CBOE Index, we assume that the dividend yield on this share equals that on the S&P 500.[13] We

[11] These calculations are based on the estimates reported on page 25 of French et al.

[12] Henry Latané and Richard Rendleman (1976) compute implied standard deviations for a series of options over a 39-week period and discover that these implied standard deviations tend to move together. Richard Schmalensee and Robert Trippi (1978) find similar results. These coincident movements in volatility are the market-wide volatility shifts we hope to capture.

[13] We assumed that dividends were paid as a continuing flow at rate λ per year, where λ is the current yield on the S&P 500.

follow Black's (1975) suggestion for dividend adjustment and subtract the present value of dividend payments over the life of the option from the price of the stock. We assume that the option on the representative stock is priced according to the formula derived by Black and Myron Scholes (1973) and apply a numerical search algorithm to determine the variance of returns which is consistent with the observed option price, risk-free rate, and market dividend yield.

Table 3 shows the estimated autocorrelogram and partial autocorrelogram for the implied volatility series. These are *weekly* data, and so the estimated first-order autocorrelation (.965) is higher than those in the last section.[14] The partial autocorrelogram once again suggests that an AR(1) model is an appropriate representation for the series. One year after a shock to volatility, expected volatility is predicted to exceed its mean by only $.965^{52} = .16$, or 16 percent, of the initial shock. The corresponding value of the elasticity of share prices with respect to volatility, calculated by rescaling the parameters α, r, and g to correspond to weekly values, is -0.041. This is roughly comparable to our estimates in the last section, and confirms our earlier conclusion that volatility changes do not persist. These data also support our claim that volatility is a stationary series, since we again reject the null hypothesis of a unit root. The test statistic for the nonstationarity hypothesis, maintaining a first-order autoregressive model, was -3.04, while the .05 critical value from Fuller is -2.88.[15]

The CBOE Call Option Index data provide some evidence on the beliefs of market participants, but they do not permit us to directly investigate how long-term expectations of volatility respond to changing short-term volatility expectations. A second source of option data can illuminate this issue. Since 1979, Value Line has computed indices of option premia at three- and six-month maturities. The availability of two different maturity option indices provides an opportunity for additional tests of the persistence hypothesis. We inverted these option premia indices using the same procedure as for the CBOE data.[16]

TABLE 3—AUTOCORRELOGRAM FOR VOLATILITY FORECASTS IMPLIED BY OPTION PREMIA

Lag Length (weeks)	Autocorrelation	Partial Autocorrelation
1	0.965(0.012)	0.965(0.012)
2	0.932(0.017)	−0.057(0.046)
3	0.895(0.021)	−0.094(0.046)
4	0.859(0.024)	−0.012(0.046)
5	0.823(0.027)	−0.061(0.046)
6	0.789(0.029)	−0.019(0.046)
7	0.753(0.031)	−0.056(0.046)
8	0.716(0.033)	−0.036(0.047)
9	0.685(0.034)	−0.031(0.047)
10	0.655(0.035)	0.017(0.047)
11	0.632(0.036)	0.096(0.047)
12	0.613(0.037)	0.034(0.047)
13	0.596(0.038)	0.039(0.047)
14	0.574(0.039)	−0.117(0.047)
15	0.551(0.039)	−0.008(0.047)
16	0.534(0.040)	0.099(0.047)
17	0.515(0.041)	−0.024(0.047)
18	0.497(0.041)	0.016(0.047)
19	0.484(0.042)	0.057(0.047)
20	0.472(0.042)	0.046(0.048)
21	0.464(0.042)	0.038(0.048)
22	0.458(0.042)	0.038(0.048)
23	0.450(0.042)	−0.039(0.048)
24	0.443(0.043)	−0.003(0.048)

Notes: Volatility forecasts are calculated by inverting the Black-Scholes option valuation formula to obtain the volatility implied by CBOE option premia indices. These data span the period 1976:1 to 1984:26, a total of 447 weekly observations. Standard errors are shown in parentheses.

[14] Because each weekly observation on the CBOE Index depends on forecasts of volatility for each of the next 26 weeks, two consecutive observations on the implied volatility will have 25 weeks of forecast volatilities in common. This may bias our estimated autocorrelations. We therefore estimated autoregressive models using nonoverlapping data periods, corresponding to every twenty-sixth observation in our data set. The estimated 6-month autocorrelation coefficient from these data is .43, which is only slightly lower than the 6-month autocorrelation implied by our weekly estimates.

[15] We also performed unit root tests with much longer autoregressive polynomials and continued to reject the unit root hypothesis at very high confidence levels.

[16] The Value Line data were available for the period 1980:16 to 1984:26; this constitutes a total of 220 weeks. However, there were 17 weeks of missing data. This precluded calculating the long autocorrelograms which are reported for the other volatility series. How-

The implied volatility for the six-month options is assumed to equal the average of the expected three-month volatilities for the next three months, and the three months following them. This means:

$$(16) \qquad {}_{t,6}\sigma_t^2 = \left({}_{t,3}\sigma_t^2 + {}_{t,3}\sigma_{t+13}^2\right)/2,$$

where ${}_{s,k}\sigma_t^2$ is the volatility expected to prevail, as of time t, over the k months beginning in week s. The assumption in (16) allows us to solve for an estimate of the implied *forward volatility* that is expected to prevail for the three-month period beginning three months from the current week:

$$(17) \qquad {}_{t+13,3}\hat{\sigma}_t^2 = 2_{t,6}\hat{\sigma}_t^2 - {}_{t,3}\hat{\sigma}_t^2.$$

Regression estimates of the change in this implied forward volatility which occurs when the current three-month "spot" implied volatility changes are

$$(18) \quad {}_{t+13,3}\hat{\sigma}_t^2 - {}_{t+12,3}\hat{\sigma}_{t-1}^2 = -.0233 + 0.511[{}_{t,3}\hat{\sigma}_t^2 - {}_{t-1,3}\hat{\sigma}_{t-1}^2]. \quad (.0769) \quad (0.050)$$

These results indicate that when current volatility expectations change, expected volatility in future periods also changes, but by much less than the change in current volatility. They imply that the half-life of a volatility shock is just over three months, and provide further evidence for our contention that volatility shocks do not persist.

IV. Conclusion and Implications

Our findings suggest that shocks to stock market volatility do not persist for long periods. Estimates based on both actual and *ex ante* volatilities indicate that these volatility shocks have half-lives of less than six months, and in some cases as short as one month. Moreover, our results suggest that substantially greater half-lives for volatility shocks can be rejected at extremely high confidence intervals. Our highest estimate of volatility persistence, obtained by fitting an IMA(1,3) process to volatilities for the 1930–84 period, suggests that the elasticity of the market price with respect to a volatility shock is −.23. Most of our estimates suggest that this elasticity is much smaller, between −.02 and −.05. These estimates imply that a doubling of volatility, which is a large shock by historical standards, would therefore reduce the level of share prices by at most 23 percent and probably by much less.

These results cast doubt on the hypothesis advanced by Malkiel and Pindyck that the transition to periods of higher uncertainty was responsible for the very poor performance of the stock market during the 1970's. They lead us to doubt that volatility fluctuations, and the movements in equity risk premia which they induce, can explain a large fraction of the variation in the stock market's level. More generally, these results only deepen the puzzle of accounting for the fundamental factors which explain the dramatic fluctuations in stock prices.

ever, the first-order autocorrelations for these series, .88 for the 3-month implied volatility and .87 for the 6-month, were roughly consistent with our earlier findings.

REFERENCES

Black, Fischer, "Fact and Fantasy in the Use of Options," *Financial Analysts Journal*, July-August 1975, *31*, 36–41, 61–72.

_____, "Studies of Stock Price Volatility Changes," in *Proceedings of the 1976 Meetings of the American Statistical Association, Business and Economic Statistics Section*, Washington: American Statistical Association, 1976, 177–181.

_____ **and Scholes, Myron,** "The Pricing of Options and Corporate Liabilities," *Journal of Political Economy*, May-June 1973, 81, 637–659.

Brainard, William, Shoven, John and Weiss, Laurence, "The Financial Valuation of the Return to Capital," *Brookings Papers on Economic Activity*, 2: 1980, 453–502.

Christie, Andrew, "The Stochastic Behavior of Common Stock Variances: Value Leverage, and Interest Rate Effects," *Journal of*

Financial Economics, December 1982, *10*, 407–32.

Dickey, David and Fuller, Wayne, "Likelihood Ratio Statistics for Autoregressive Time Series with a Unit Root," *Econometrica*, July 1981, 49, 1057–1072.

French, Kenneth, Schwert, G. William and Stambaugh, Robert, "Expected Stock Returns and Volatility," Working Paper MERC 85–10, Graduate School of Management, University of Rochester, 1985.

Fuller, Wayne, *The Statistical Analysis of Time Series*, New York: Wiley & Sons, 1976.

Ibbotson, Roger G., *Stocks, Bonds, Bills, and Inflation: 1984 Yearbook*, Chicago: R. G. Ibbotson Associates, 1984.

Kleidon, Allan, "Variance Bounds Tests and Stock Price Valuation Models," Research Paper 806, Stanford University Graduate School of Business, 1985.

Latané, Henry and Rendleman, Richard, Jr., "Standard Deviations of Stock Price Ratios Implied in Option Prices," *Journal of Finance*, May 1976, *31*, 369–81.

Malkiel, Burton, G., "The Capital Formation Problem in the United States," *Journal of Finance*, May 1979, 34, 291–306.

Mankiw, N. Gregory, Romer, David and Shapiro, Matthew, "An Unbiased Reexamination of Stock Market Volatility," *Journal of Finance*, May 1985, *40*, 677–87.

Marsh, Terry A. and Merton, Robert C., "Dividend Variability and Variance Bounds Tests for the Rationality of Stock Market Prices," *American Economic Review*, June 1986, *76*, 483–98.

Merton, Robert C., "An Intertemporal Capital Asset Pricing Model," *Econometrica*, September 1973, *41*, 867–87.

_____, "On Estimating the Expected Return on the Market," *Journal of Financial Economics*, December 1980, *8*, 323–61.

Officer, Robert R., "The Variability of the Market Factor of the New York Stock Exchange," *Journal of Business*, July 1973, *46*, 434–53.

Pindyck, Robert S., "Risk, Inflation, and the Stock Market," *American Economic Review*, June 1984, *74*, 335–51.

Schmalensee, Richard and Trippi, Robert, "Common Stock Volatility Expectations Implied by Option Premia," *Journal of Finance*, March 1978, *33*, 129–47.

Schwert, G. William, "Effects of Model Specification on Tests for Unit Roots," Working Paper GPB 85-10, Economics Department, University of Rochester, 1985.

Shiller, Robert, "Do Stock Prices Move Too Much to be Justified by Subsequent Changes in Dividends?," *American Economic Review*, June 1981, *71*, 421–36.

Chicago Board Options Exchange, *The CBOE Call Option Index: Methodology and Technical Considerations*, Chicago, 1979.